Consumers' Privacy Choices in the Era of Big Data*

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Abstract

While consumers often feel overwhelmed by the complexity involved in choices regarding personal data, sellers with superior information processing algorithms are enabled to make more tailored offers in times of increasing datafication. We construct a model where consumers are confronted with a seller whose big data algorithms extract surplus via customized pricing. They face a trade-off between a direct, transaction cost-free sales channel and a privacy-protecting, but costly, channel when buying a product. We show that the privacy-protecting channel is used even in the absence of an explicit taste for privacy if consumers are not *too* strategically sophisticated, thereby microfounding privacy preferences.

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"Few consumers have ever heard of ***Acxiom***. But analysts say it has amassed the world's largest commercial database on consumers — and that it wants to know much, much more. Its servers process more than 50 trillion data 'transactions' a year. Company executives have said its database contains information about 500 million active consumers worldwide, with about 1,500 data points per person. That includes a majority of adults in the United States."

(The New York Times 2012)

Shopkick offers a smartphone app that rewards users for checking into stores, scanning products, visiting the dressing rooms, and so forth. Founded in 2009, it has not only brought more than USD 1 billion in revenues for its corporate partners via generating more than 50 million walk-ins to partner stores and 100 million product scans (Shopkick 2014); with over 6 million users spending more than 3 hours per month it is also the most-used shopping app, according to Nielsen (2012).

(Shopkick 2014 and Nielsen 2012)

Amazon recently was issued a patent on a novel *Method and System for Anticipatory Package Shipping.*[1] "So Amazon says it may box and ship products it expects customers in a specific area will want – based on previous orders and other factors – but haven't yet ordered. [...] [T]he patent demonstrates one way Amazon hopes to leverage its vast trove of customer data to edge out rivals. [...] Based on all the things they know about their customers they could predict demand based on a variety of factors."

(Wall Street Journal Blog 2014)

These contemporary business cases exemplify two recent technological developments. On the one hand side, firms get better in drawing relevant information about certain groups of people or even individuals out of huge data sets. On the other side, such data sets are increasingly available, owing to the fact that more economic and social transactions take place aided by information and communication technologies, which easily and inexpensively store the information they produce or transmit. Taken together, these developments constitute the rise of big data (Mayer-Schönberger and Cukier 2013).

[1]Patent number US008615473 (December 24, 2013), `http://pdfpiw.uspto.gov/.piw?docid=08615473`

They imply that sellers can make consumers ever more tailored contract offers, which fit their individual preferences or consumption patterns.[2]

One logical consequence of this process is that sellers can approximate first-degree price discrimination better than ever before. First-degree (or perfect) price discrimination is characterized by complete information of a seller about a specific consumer's willingness-to-pay for a certain product (Pigou 1920). This information leads to the seller's ability to appropriate all surplus of the transaction, assuming that reselling is impossible or unprofitable, because he can set a price that just equals the consumer's valuation of the product. However, due to the very high information demand of the seller about consumers' preferences and the rather straightforward allocative and distributional implications, perfect price discrimination has not received a lot of attention in the economics literature and has mostly been dismissed as a mere theoretical construct.[3]

More prominent are models of so-called "behavior-based price discrimination." Most of this literature focuses on second-degree price discrimination by assuming that a seller learns about the willingness-to-pay of a re-identifiable or recognizable consumer after the first purchase of a good. The idea is that, if a consumer previously bought a product at a certain price, the seller would learn that this particular consumer's willingness-to-pay must have exceeded the price for which she bought the product.[4] However, as the introductory examples above illustrate, online vendors and other retailers have already gone much further and can approximate fully personalized prices more than ever, which supports the early conclusion of Odlyzko, "that in the Internet environment, the incentives towards price discrimination and the ability to price discriminate will be growing" (Odlyzko 2003, 365).

It has been shown empirically that "targeted advertising" techniques increase purchases (Luo et al. 2014), prices (Mikians et al. 2012), and sellers' profits (Shiller 2013). Some consumers, however, feel repelled by this development, which assigns a passive role to them, facing apparently omniscient sellers who can exploit all their digital traces.[5] Many want to have control over their personal data back.[6] Many place a value on their

[2]Such offers can be made directly, for instance, in online retailing, or indirectly, via selling advertisers access to highly preselected consumer groups. Einav and Levin (2013) provide a list of illustrative examples how firms, public administration, and researchers can exploit such novel technological opportunities.

[3]For instance, the standard industrial organization textbook, Tirole (1988), spends three of its more than 1100 pages on perfect price discrimination.

[4]For an overview of this strand of literature, see Fudenberg et al. (2006).

[5]Shiller (2013, 5) reports: "Even if consumers did understand which behaviors result in low prices, they might prefer to ignore them rather than change potentially thousands of behaviors just to receive a lower quoted price for one product."

[6]Goldfarb and Tucker (2012) study three million observations between 2001 and 2008 and find that refusals to reveal their income in an online survey have risen over time. Tucker (2014) finds in a field

privacy (Tsai et al. 2011).

The early theoretical literature about the economics of privacy, being based on the Chicago school argument that more information available to market participants increases the efficiency of markets, has underlined the negative welfare effects of hiding information from sellers (Posner 1978; Posner 1981; Varian 1997). A lot of progress in our understanding has been made since then.[7] Most articles have focused on the choices of firms that own some type of personal information about consumers and can decide to disclose it to another firm, or not (Taylor 2004; Acquisti and Varian 2005; Calzolari and Pavan 2006; Casadesus-Masanell and Hervas-Drane 2015). The core question studied in these papers is, what the welfare consequences of privacy or disclosure are, and who should own the property rights of consumers' personal data (Hermalin and Katz 2006).[8] The answers given have been ambiguous and depend on the specific application of the papers. Recently, the focus has shifted more towards the privacy choices of consumers (Conitzer et al. 2012) and to the role of platform intermediaries (de Corniere and De Nijs 2014). [9]

Regarding these choices, Acquisti and Grossklags (2007, 369) note: "Consumers will often be overwhelmed with the task of identifying possible outcomes related to privacy threats and means of protection. [...] However, even if individuals had access to complete information, they would often be unable to process and act optimally on large amounts of data. Especially in the presence of complex, ramified consequences associated with the protection or release of personal information, our innate bounded rationality limits our ability to acquire, memorize and process all relevant information, and it makes us rely on simplified mental models, approximate strategies, and heuristics. Bounded problem

experiment that, when Facebook gave users more control over their personally identifiable information, users were twice as likely to click on personalized ads. Brandimarte et al. (2012, 6) conclude: "Three experiments provide empirical evidence that perceived control over release plays a critical role in sharing/oversharing personal information, relative to the objective risks associated with information access and usage by others."

[7] Already Hermalin and Katz (2006, 229) made clear: "With so many people making extreme claims in discussions of privacy and related public policy, and with so little understanding of the underlying economics, it is important to identify the fundamental forces clearly. A central fact is that, contrary to the Chicago School argument, the flow of information from one trading partner to the other can reduce ex post trade efficiency when the increase in information does not lead to symmetrically or fully informed parties."

[8] On the Internet, for instance, the customer databases of sellers or intermediaries, such as search engines, tracing back the physical address of users on the basis of their IP address (or to clearly identify them as persons on the basis of their registration data or a unique identifier derived from a permanent cookie) was recently qualified as "personal data" (Opinion 1/2008 on Data Protection Issues Related to Search Engines, Advisory Working Party (adopted Apr. 4, 2008) (EC), Data Protection available at `http://ec.europa.eu/justice/policies/privacy/docs/wpdocs/2008/wp148\_en.pdf`).

[9] For an overview of this strand of literature, see Acquisti et al. (2016).

solving is usually neither unreasonable nor irrational, and it doesn't need to be inferior to rational utility maximization." With few exceptions, however, cognitive constraints of consumers have not been incorporated by theoretical studies of markets driven by big data.[10]

In this paper, we account for the difference between consumers with limited cognitive sophistication and sellers, whose data analysis capabilities outperform consumers' capabilities by far. Even if perfect price discrimination was not feasible yet today, it constitutes the limit case which developed economies are approaching. Therefore, combining two core concepts of industrial organization (perfect price discrimination and monopoly markets) with cognitive limitations, we ask: What are the effects of perfect price discrimination on equilibrium behavior and welfare when consumers' anonymization is possible but costly? How are these effects affected by the level of strategic sophistication of consumers?

The main contribution of this paper is to show under which circumstances a costly privacy-protective sales channel can exist even when consumers do not have an explicit taste for privacy and how this equilibrium depends on consumers' level of sophistication. We thereby provide a micro-foundation for such privacy preferences.

We construct a model where a mass of consumers with heterogeneous willingness-to-pay for a product is facing a monopolistic seller who can be approached via two sales channels.

The direct channel (D) makes use of all personal information that the seller has about every single consumer, be it via collecting such information in the past (e.g. Amazon) or via buying such information from an intermediary (e.g. Google, Acxiom). We assume that the seller can perfectly discriminate prices when selling through channel D and that this channel is transaction cost-economizing, in the sense that a consumer can shop without bearing additional cost.

The anonymous channel (A) protects consumer privacy by anonymizing the search choices of individual users and does not track their behavior (e.g. Ixquick/Startpage, DuckDuckGo, or shopping offline with cash). As a consequence, perfect price discrimination is infeasible for the seller, who responds best by setting a uniform price for this channel. The disadvantage for using channel A can stem from decreasing precision of product search results compared to channel D, which results in extra time or effort that

[10]Taylor (2004), Acquisti and Varian (2005), and Armstrong (2006) assume the existence of a group of perfectly rational consumers and a group of naïve consumers. The latter do not foresee that they may want to trade in the future again and, because of this negligence, ignore the negative effects of disclosing personal data. In our model, we allow for a more nuanced, marginal analysis of consumers' sophistication.

a consumer must spend to find her preferred product.[11] Moreover, using this channel might require the consumer to change privacy settings, install potentially costly privacy-protective software, or use other privacy-enhancing techniques.

Consumers can decide which channel to use to buy the product. The seller can neither directly influence consumers' channel choice nor close down the anonymous channel as the anonymization technique is at consumers' disposal and he might simply not be allowed to not serve anonymized consumers.

In the three stages of our model, the consumers first choose between channel D and channel A. Second, the seller sets prices in both channels. Finally, every consumer decides whether to buy for the price offered to her, or not. As our analysis is based on a model of limited strategic thinking, called *level k-thinking*, we solve this game by backward induction for a *Sophistication-k equilibrium* (defined in Binswanger and Prüfer (2012) and related to a Perfect Bayesian Equilibrium, just with level k-beliefs). The underlying model was introduced by Stahl and Wilson (1994; 1995) and Nagel (1995). Since then, a sizeable literature has developed that explores k-thinking theoretically and empirically, including Ho et al. (1998), Costa-Gomes et al. (2001), Crawford (2003), Camerer et al. (2004), Costa-Gomes and Crawford (2006), Crawford and Iriberri (2007a), Goldfarb and Yang (2009), and Binswanger and Prüfer (2012), among others. The literature has found strong experimental support for level k-thinking and suggests values for k of one or two (Camerer, et al. 2004; Crawford and Iriberri 2007b).

We model consumers' cognitive constraints by their ability to anticipate k strategic iterations, nesting the usual assumptions in the behavior-based price discrimination literature of either unlimited strategic sophistication (corresponding to $k = \infty$) or complete naïveté of consumers (corresponding to $k = 0$).[12] The seller, on the other hand – due to superior access to data and computing power – is able to outperform consumers in strategic thinking (i.e. has a level of at least $k + 1$) and hence always employs the optimal response to consumers' strategies. Whether k is to be seen as a low number, as suggested by the empirical behavioral literature, or rather infinitely high turns out to crucially matter for our results.

We show that the higher consumers' degree of sophistication, the *higher* the equilibrium price will be on the anonymized market of channel A. If consumers are highly sophisticated, they anticipate the incentive of the seller to increase the price beyond the

[11] See the literature cited in Argenton and Prüfer (2012) documenting the effect from access to more search log data on the quality of search engines as perceived by users.

[12] What we call "unlimited strategic sophistication", is often referred to as "perfect rationality". However, players with limited strategic sophistication still act rationally given their (potentially imperfectly updated) beliefs, which is why we avoid the terms of "perfect" and "imperfect" rationality.

price expected by less sophisticated consumers. Therefore, if the level of sophistication rises in the population, more consumers (those with medium but not high willingness-to-pay) will expect to be offered the product for a high price on the anonymized market—and hence choose sales channel D at Stage 1, preempting net losses. Consequently, the seller has an incentive to increase the price in the anonymized market even more because he infers that only consumers with high willingness-to-pay have chosen channel A at Stage 1.

We further show that, with any positive cost of anonymization, the anonymized market completely unravels for all sophistication levels $k \geq \bar{k}$, where $\bar{k}$ is a finite number (and hence also covers the case of $k = \infty$, where consumers have unlimited strategic sophistication). If consumers have binding cognitive constraints, however, only a part of the market unravels and the anonymized sales channel can persist, serving consumers with relatively high willingness-to-pay. Among those who use the privacy-protecting sales channel, some consumers suffer from net losses because prices turn out to be higher than expected, but consumers with a very high willingness-to-pay get some surplus. Thereby, this model offers a micro-foundation of consumers' privacy preferences, as consumers can rationally want to use anonymization techniques (or their data erased) even without an exogenous taste for privacy (with such an exogenous taste for privacy, our results are reinforced). Because the anonymized sales channel is used in equilibrium and a share of the anonymization cost s could be interpreted as a (royalty) fee that an intermediary can appropriate, this model also suggests that running a consumer privacy-protecting sales channel competing with a channel that tracks individuals and uses all personal data can be a sustainable business model in an economy populated by consumers with limited strategic sophistication.

1 Model

1.1 Setup

We consider an economy where a monopolistic seller of a single consumption good faces a continuum of consumers who can buy at most one unit of the good and cannot resell it to each other. Abstracting from potential fixed costs, we assume that the monopolist can produce the good at constant marginal cost $c \geq 0$ and consumers have a heterogeneous valuation v_i for the good, where $v_i \sim U[0, 1]$. Outside options yield zero payoff and in case of indifference consumers prefer buying the good to not buying the good. We assume that consumers can buy the good from the seller in two different ways: using a direct sales channel (referred to as channel D) or making use of an anonymization technique (referred to as channel A). If a consumer chooses direct channel D, the seller perfectly knows her valuation (i.e. her maximum willingness-to-pay) through his large customer information database. However, if a consumer chooses channel A her valuation is hidden from the selle. Using the anonymization technique comes at cost $s > 0$. This cost can reflect various sources of disutility ranging from a monetary payment for an anonymization service or lower internet connection speed resulting from using anonymization tools, a decrease in product search quality, or additional transaction costs if the anonymization technique is "shopping offline". The seller can neither directly influence consumers' channel choice nor close down channel A. The distribution of v_i, the cost for anonymization s, the monopolist's cost structure (and hence the supply function) as well as the timing of the game are common knowledge among all players.

Further, we assume that all players are solely interested in their own material payoff (i.e. net monetary profit or consumer surplus). Specifically, consumers do not have any exogenous taste for privacy (and even consumers that are indifferent between channel A and channel D will choose the direct channel rather than the anonymous one). To account for the cognitive constraints that consumers face, we assume that all consumers have a certain, homogenous level of strategic sophistication, denoted by k. As a reflection of the seller's higher analytical capacities (due to his computing power, information databases and forecasting algorithms) we assume that the seller's level of strategic sophistication is always higher than the level of the consumers. Suppose the level of strategic sophistication of the consumers is given by k, then the seller will have a level of at least $k + 1$. The seller could also be modeled as having unlimited strategic sophistication, but for our results the weaker assumption of the seller being more strategically sophisticated than the consumers is already sufficient.

Our model therefore describes the situation *after* a long period during which consumers have not used anonymization techniques and marketers have collected data shedding light on individual consumers' preferences. The timing of the model is as follows:

- Stage 1 (*Anonymizing*): Based on their valuation v_i and their price expectations for both channels, consumers choose between channel D or channel A and incur costs of 0 or s, respectively. Indifferent consumers choose channel D.

- Stage 2 (*Pricing*): The seller sets prices $p = \{p_i, p_A\}$, where p_i are individual prices for each consumer in channel D, and p_A is the uniform price p_A for all channel A consumers.

- Stage 3 (*Buying*): Consumers decide whether to buy the good for the offered price.

Due to the limited strategic sophistication of consumers the game cannot be solved for a Perfect Bayesian equilibrium (which is nested in our analysis, though, if $k = \infty$) but for an equilibrium inspired by the level-k literature. While most of this literature deals with (fairly) symmetric decisions (e.g. the beauty contest game), we have to adapt the concept to the asymmetric situation of our model. We therefore employ an adjusted version of the *Sophistication-k Equilibrium* as defined in Binswanger and Prüfer (2012), which builds upon the definition of a Perfect Bayesian Equilibrium but adds level-k beliefs.[13]

Definition 1 (Modified Sophistication-k Equilibrium) *A sophistication-k equilibrium is a strategy combination and a set of beliefs about the behavior of the other player, such that at each node of the game between a level-k player (the consumer) and a level-k+1 player (the seller):*

1. *The strategies for the remainder of the game are Nash equilibria given the beliefs and strategies of the other player;*
2. *The level-k player holds a k-belief about the behavior of the other player;*
3. *The k+1 player anticipates the belief of the level-k player.*

The belief structure we assume for our model is therefore as follows:

Assumption 1 (Belief structure)

- *All players take into account that $v_i \in [0, 1]$ and hence that $p_i \in [0, 1]$ and $p_A \in [0, 1]$ (eliminating beliefs higher than the maximum valuation).*
- *Consumers form a belief about the prices the seller will set in channel D and channel A, denoted by $\mathbb{E}(p|D)$ and $\mathbb{E}(p|A)$, respectively.*
- *The seller forms a belief about which consumers have chosen channel D and channel A and correctly anticipates the beliefs consumers have about prices.*

Note that since consumers' level of sophistication is lower than the seller's it does not need to be the case that $\mathbb{E}(p|D) = (p|D)$ and $\mathbb{E}(p|A) = (p|A)$ when each channels is at least used by some consumers (as it would have to be in a Perfect Bayesian Equilibrium). Their cognitive limitation notwithstanding, players still employ Nash equilibrium strategies for the remainder of the game given their beliefs and the game is solved using backward induction.

[13] In contrast to Binswanger and Prüfer, there are no differing *states of nature* in our model. Hence, we do not need to condition beliefs on the state of nature and drop the fourth part of their definition stating: "The beliefs about the state of nature are rational and determined by Bayes' rule."

2 Analysis

Stage 3 – Buying: A utility-maximizing consumer decides to buy the product if, and only if, the price she has to pay for it does not exceed her valuation of the good, i.e. if, and only if,

$$v_i \geq p \in \{p_i, p_A\}. \tag{1}$$

If she has chosen for channel A, the price will be an individualized price p_i, and if she has chosen for channel D, she will receive the same uniform price p_A as all other consumers who have chosen channel D.

Stage 2 – Pricing: A profit-maximizing seller sets individual prices p_i for all consumers approaching him via channel D (denoted by set $\mathcal{C}_D$) and one optimal uniform price p_A for all anonymized consumers in channel A (denoted by set $\mathcal{C}_A$). Knowing v_i precisely for all consumers in $\mathcal{C}_D$, the seller trivially sets

$$p_i^* = \max\{v_i, c\} \text{ for all } i \in \mathcal{C}_D, \tag{2}$$

where the lower bound c takes into consideration that it is not optimal to sell below marginal cost. Being uninformed about the individual valuations v_i of all consumers in $\mathcal{C}_A$ the seller can nevertheless infer which consumers are in $\mathcal{C}_A$, due to his superior strategic sophistication, and set p_A accordingly. We will therefore analyze consumers' general Stage 1 behavior first in order to inform the seller's pricing decision in channel A.

Stage 1 – Anonymizing: Consumers will choose to use the anonymization technique of channel A if, and only if, the expected utility of doing so exceeds the expected utility of the direct channel D, i.e. if, and only if, $\mathbb{E}(u_i(A)) > \mathbb{E}(u_i(D))$, where

$$\mathbb{E}(u_i(D)) = \max\{v_i - \mathbb{E}(p|D), 0\}, \tag{3}$$

$$\mathbb{E}(u_i(A)) = \max\{v_i - \mathbb{E}(p|A) - s, -s\}. \tag{4}$$

The first value in each set reflects the payoff the consumer were to receive should she choose to buy the product at Stage 3. The second value reflects the payoff of subsequently choosing not to buy the product. Although consumers might be limited in their strategic sophistication, we will nonetheless assume that they understand the nature of the two channels, i.e. they understand that the seller has no incentive to decrease the price below their valuation in channel D and that the seller can only ask for a uniform price in channel A. Hence, consumers form the price expectation for channel D

$$\mathbb{E}(p|D) = p_i^* = \max\{v_i, c\}, \tag{5}$$

irrespective of their level of strategic sophistication k and therefore correctly expect to be left with no surplus when choosing channel D. With respect to channel A, however, consumers only know for sure that the seller has to set a uniform price. Which price exactly they expect depends on their level of strategic sophistication. For now, it is sufficient to replace the expectation $\mathbb{E}(p|A)$ by the expectation of a single uniform price $\mathbb{E}(p_A)$:

$$\mathbb{E}(u_i(D)) = \max\{v_i - \max\{v_i, c\}, 0\} = 0, \tag{6}$$

$$\mathbb{E}(u_i(A)) = \max\{v_i - \mathbb{E}(p_A) - s, -s\}, \tag{7}$$

Comparing these expected payoffs shows that consumers will choose channel A if, and only if,

$$\max\{v_i - \mathbb{E}(p_A) - s, -s\} > 0. \tag{8}$$

Since $s > 0$, this can only hold if $v_i > \mathbb{E}(p_A) + s \equiv \hat{v}$, where $\hat{v}$ denotes the endogenous threshold dividing the population of consumers into $\mathcal{C}_D$ and $\mathcal{C}_A$.

Lemma 1 (Anonymization Threshold) *There exists a threshold $\hat{v} = \mathbb{E}(p_A) + s$ that denotes the valuation of a consumer who is indifferent between both channels at Stage 1. Consumers with $v_i > \hat{v}$ will prefer channel A to channel D; consumers with $v_i \leq \hat{v}$ prefer channel D to channel A, i.e. $\mathcal{C}_D = [0, \hat{v}]$ and $\mathcal{C}_A = (\hat{v}, 1]$.*

Stage 2 – Pricing (revisited): Having a higher level of strategic sophistication than the consumers, the seller correctly infers $\hat{v}$ and hence knows that $\mathcal{C}_A = (\hat{v}, 1]$. As he further anticipates that consumers will buy the product at Stage 3, if, and only if, $v_i \geq p_A$, he can easily infer demand $q_A(p_A)$ in channel A:

$$q_A(p_A) = \begin{cases} 0 & \textit{if } p_A > 1, \\ 1 - p_A & \textit{if } 1 \geq p_A > \hat{v}, \\ 1 - \hat{v} & \textit{if } \hat{v} \geq p_A. \end{cases} \tag{9}$$

It becomes immediately clear that charging $p_A = \hat{v}$ dominates all prices $p'_A < \hat{v}$ as any price below $\hat{v}$ only leads to less profits per unit sold without an increase in quantity to counter the loss. Thus by setting $p_A = \hat{v}$, the seller can guarantee himself profits from channel A of:

$$\pi_A(\hat{v}) = q_A(\hat{v})(\hat{v} - c) = (1 - \hat{v})(\hat{v} - c). \tag{10}$$

However, the seller might still find it optimal to charge a price $p_A > \hat{v}$, depending on where $\hat{v}$ lies exactly. Suppose for the moment that the entire consumer population uses channel A (i.e. $\hat{v} = 0$), which is identical with the case of a monopolist that cannot engage in price discrimination and let us denote the globally profit-maximizing price in this case by p_M which is given by $p_M = \frac{1+c}{2}$. Then, there are three different cases for the location of the anonymization threshold $\hat{v}$ (shown in Figure 1) compared to p_M:

(a) The anonymization threshold is below the standard monopoly price ($\hat{v} < p_M$).

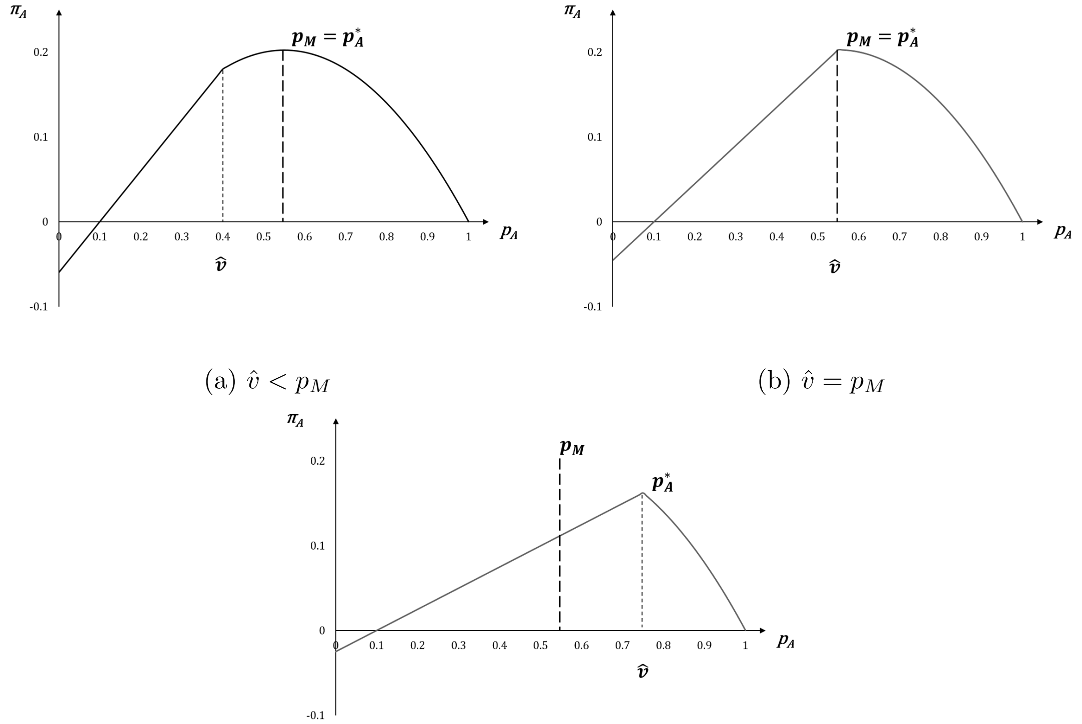
(b) The anonymization threshold is equal to the standard monopoly price ($\hat{v} = p_M$).

(c) The anonymization threshold is above the standard monopoly price ($\hat{v} > p_M$).

In cases (a) and (b), the globally profit-maximizing price p_M is in the support of the demand function and hence remains the optimal price to set. The only consumers that are not in $\mathcal{C}_A$ are those that the seller would not have served even if they had anonymized themselves. Only in case (c), where the globally profit-maximizing price p_M is not in the support anymore, the previous argument applies that a price below the anonymization threshold $\hat{v}$ is at least dominated by setting the price equal to this threshold. The seller also has no incentive to raise the price above $\hat{v}$ as profits are strictly decreasing to either side of the global maximum at p_M due to the strict concavity of the profit function. Hence, in this case the optimal price p_A^* is equal to $\hat{v}$. The seller's complete optimal pricing strategy for both channels is summarized in Lemma 2.

Lemma 2 (Optimal Pricing Strategy) *The optimal pricing strategy of the seller consists of a set of prices $\{p_i^*, p_A^*\}$ charged in channel D and channel A, respectively, where $p_i^* = \max\{v_i, c\}$ and $p_A^* = \max\{\hat{v}, p_M = \frac{1+c}{2}\}$.*

Note that this optimal pricing strategy implies that the seller sets a higher price than consumers had expected whenever $p_A^* = \hat{v}$ as $\hat{v} = \mathbb{E}(p_A) + s > \mathbb{E}(p_A)$. This is due to the fact that s will be a sunk cost for consumers at Stage 3, which the seller can anticipate and exploit via increasing the price by exactly s, compared to their expectations. Consumers,



(a) $\hat{v} < p_M$

(b) $\hat{v} = p_M$

(c) $\hat{v} > p_M$

Figure 1: Profits in channel A for different locations of $\hat{v}$ with parameters $v_i \sim \mathcal{U}[0, 1], c = 0.1$

on the other hand, do not anticipate this price increase due to their limited strategic sophistication, which influences their expectation formation.

Stage 1 – Anonymizing (revisited): The last missing piece to fully characterize equilibrium behavior is the formation of consumers' expectations of the price in channel A $\mathbb{E}(p_A)$ in Stage 1. As outlined earlier, we describe this by level-k thinking, which is best determined recursively. Thus, we will start with the case of consumers with a strategic level of sophistication of $k = 0$, which are referred to as "naïve" consumers. They naïvely expect the monopolist to engage in regular monopoly pricing[14] in channel A, i.e. $\mathbb{E}_0(p_A) = p_M$, ignoring the fact that the very choice of channel A might be signaling a high willingness to pay to the seller. For channel D, we have already assumed

[14] Alternative assumptions about starting points for naïve consumers are discussed in Section 4.1. We chose in favor of expositional simplicity.

that even the most naïve (but still rational) consumer foresees perfect price discrimination in channel D as it does not require iterative thinking. The equilibrium behavior if consumers are completely naïve is summarized in Lemma 3.

Lemma 3 (Sophistication-0 Equilibrium) *For any non-prohibitively high cost of anonymization $s > 0$ and cost of production $c > 0$ it holds that: If consumers are strategically "naïve", there is a unique sophistication-0 equilibrium with the following characteristics:*

- *Consumers form the 0-beliefs $\mathbb{E}_0(p_D) = p_i^*$ and $\mathbb{E}_0(p_A) = p_M = \frac{1+c}{2}$.*
- *Consumers anonymize if, and only if, $v_i > \hat{v}_0 = p_M + s$, separating into the sets $\mathcal{C}_D = [0, \hat{v}_0]$ and $\mathcal{C}_A = (\hat{v}_0, 1]$.*
- *The seller forms the 1-belief that $\mathcal{C}_D = [0, \hat{v}_0]$ and $\mathcal{C}_A = (\hat{v}_0, 1]$.*
- *The seller sets $p_i^* = \max\{v_i, c\}$ and $p_{A_0}^* = \hat{v}_0 = p_M + s$.*
- *All consumers in $\mathcal{C}_D$ with $v_i \geq c$ buy the product at the price offered to them.*
- *All consumers in $\mathcal{C}_A$ buy the product at the price offered to them.*

Lemma 3 shows that consumers in channel A pay a surcharge of s as compared to their expectations ($p_{A_0}^* - \mathbb{E}_0(p_A) = s$). Due to their cognitive constraints, consumers do not foresee that, once they reach Stage 3, the anonymization cost s will have turned into a sunk cost. The seller can exploit this fact because he knows that only consumers with a valuation of at least $p_M + s$ choose the anonymous sales channel. Given this lower bound on the valuations in $\mathcal{C}_A$, the seller can ignore that anonymized consumers spent s on top, and extract the lower bound's full willingness-to-pay. This divergence between expected price and realized price, in turn, informs us about the way in which consumers form their price expectation for higher levels of strategic sophistication, $k > 0$.

If instead of being naïve ($k = 0$), all consumers had a strategic sophistication level of $k = 1$, they were capable of one iteration of strategic reasoning and hence anticipate that seller's the best response to the 0-belief is to set $p_{A_0}^* = p_M + s$. Therefore, they adjust their expectation and accordingly form the 1-belief $\mathbb{E}_1(p_A) = p_{A_0}^* = p_M + s$ leading to $\hat{v}_1 = p_M + 2s$, to which the seller's best response is $p_{A_1}^* = p_M + 2s$ (analogue to the reasoning above). This in turn would be anticipated if all consumers had a strategic sophistication level of $k = 2$ forming the 2-belief $\mathbb{E}_2(p_A) = p_{A_1}^* = p_M + 2s$, and so forth.

More generally we can write, for any level of k:

$$\mathbb{E}_k(p_A) = p_M + ks, \tag{11}$$

$$p^*_{A_k} = p_M + (k+1)s = \hat{v}_k. \tag{12}$$

Thus, at every additional level of strategic sophistication, consumers will incorporate the sunk cost one more time than at the previous level, which induces the seller to raise the price once more. Consequently, $\hat{v}_k$ is increasing in k, in turn causing $\mathcal{C}_A$ to shrink in size as k increases. The more strategically sophisticated the population of consumers is, the fewer consumers will choose to anonymize until a point is reached where no consumer does so anymore. Then, channel A remains unused and the market for anonymization breaks down completely. This point is reached when the anonymization threshold matches or exceeds even the highest willingness-to-pay of any due to the beliefs consumer form about p_A. We denote the threshold level of strategic sophistication from which onwards this is the case by $\bar{k}$ and define:

$$\bar{k} \equiv \min\{k \in \mathbb{Z}^+_0 | \hat{v}_k \geq 1\}. \tag{13}$$

The inequality condition of Equation (13) can hold with equality as any consumer indifferent between the two channels opts for channel D, including the one with the maximum valuation for the good $v_i = 1$. Using Equation (12) in (13) and solving for $\bar{k}$ yields:

$$\bar{k} \geq \frac{1-c}{2s} - 1. \tag{14}$$

In the case where $\frac{1-c}{2s} - 1$ is an integer, Equation (14) holds with equality and $\bar{k}$ is identified. In the case where $\frac{1-c}{2s} - 1$ is not an integer, we have to round to the next higher integer $\lceil \frac{1-c}{2s} - 1 \rceil$ to fulfill the requirement that $\bar{k}$ be an integer. Recalling that $\lceil x \rceil = x$ for $x \in \mathbb{Z}$, we can combine both cases to

$$\bar{k} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil, \tag{15}$$

which shows that channel A breaks down at a *finite* level of strategic sophistication. Because the iteration stops at a finite $\bar{k}$, we have implicitly solved for the limit case of $k = \infty$: If consumers had unlimited strategic sophistication, it would deliver the same result ($\mathcal{C}_A = \emptyset$) as any other $k \geq \bar{k}$. Thus, while unlimited strategic sophistication is a *sufficient* condition for a breakdown of channel A, it is *not* a *necessary* condition.

Lemma 4 (Usage of Channel A) *For any non-prohibitively high cost of anonymization $s > 0$ and cost of production $c > 0$, the anonymous channel is used if, and only if, consumers are not* too *strategically sophisticated, i.e. if $k < \bar{k} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil$.*

That channel A breaks down at a finite level of sophistication $\bar{k}$ has consequences for the belief formation of consumers when $k > \bar{k}$. While belief formation according to Equation (11) does not violate that all players take into account that $p_A \in [0,1]$ for $k \leq \bar{k}$, this is not the case for $k > \bar{k}$. Denoting any level of consumer sophistication $k > \bar{k}$ by $\bar{k}^+$, we specify beliefs $\mathbb{E}_{\bar{k}^+}(p_A)$ that meet this condition (Equation 16). Additionally, in line with Lemma 4, any belief $\mathbb{E}_{\bar{k}^+}(p_A)$ has to render the choice of channel D a Nash strategy for consumers regardless of their valuation (Equation 17). This yields:

$$\mathbb{E}_{\bar{k}^+}(p_A) \in [0;1] \qquad \Rightarrow \mathbb{E}_{\bar{k}^+}(p_A) \leq 1, \tag{16}$$

$$\hat{v}_{\bar{k}^+} = \mathbb{E}_{\bar{k}^+}(p_A) + s \geq 1 \qquad \Rightarrow \mathbb{E}_{\bar{k}^+}(p_A) \geq 1 - s. \tag{17}$$

Both conditions are satisfied for any belief $\mathbb{E}_{\bar{k}^+}(p_A) \in [1-s, 1]$. Hence, multiple beliefs are possible when $k > \bar{k}$, but all imply that channel A remains unused. For any level of consumer sophistication where channel A remains unused (including $k = \bar{k}$), the seller forms the k+1-belief that $\mathcal{C}_D = [0,1]$ and $\mathcal{C}_A = \emptyset$. Therefore, setting p_A is an off-equilibrium action and the seller can set any price $p^*_{A_{\bar{k}^+}} \in [0;1]$ (restricted only by the support of the demand function). However, in the special case of unlimited strategic sophistication (corresponding to a Perfect Bayesian Equilibrium), it has to hold that $\mathbb{E}(p|A) = (p|A)$. Hence, he sets $p^*_{A_\infty} = \mathbb{E}_\infty(p_A)$ implying that $p^*_{A_\infty} \in [1-s, 1]$.

Combining the insights of the previous Lemmas, we summarize the equilibrium analysis with the formulation of the general sophistication-k equilibrium in Proposition 1.

Proposition 1 (Sophistication-k Equilibrium) *For any non-prohibitively high cost of anonymization $s > 0$ and cost of production $c > 0$ it holds that:*

1. *If consumers have a level of strategic sophistication of $k \leq \bar{k} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil$, there is a unique sophistication-k equilibrium with the following characteristics:*

    - *Consumers form the k-beliefs $\mathbb{E}_k(p_D) = p^*_i$ and $\mathbb{E}_k(p_A) = p_M + ks = \frac{1+c}{2} + ks$.*
    - *Consumers anonymize if, and only if, $v_i > \hat{v}_k = p_M + (k+1)s$, separating into the sets $\mathcal{C}_D = [0, \hat{v}_k]$ and $\mathcal{C}_A = (\hat{v}_k, 1]$ (where $\mathcal{C}_A = \emptyset$ if $k = \bar{k}$).*
    - *The seller forms the k+1-belief that $\mathcal{C}_D = [0, \hat{v}_k]$ and $\mathcal{C}_A = (\hat{v}_k, 1]$.*
    - *If $k < \bar{k}$, the seller sets $p^*_i = \max\{v_i, c\}$ and $p^*_{A_k} = \hat{v}_k = p_M + (k+1)s$.*

- *If $k = \bar{k}$, the seller sets $p_i^* = \max\{v_i, c\}$ and $p_{A_{\bar{k}}}^* \in [0, 1]$.*
- *All consumers in $\mathcal{C}_D$ with $v_i \geq c$ buy the product at the price offered to them.*
- *All consumers in $\mathcal{C}_A$ buy the product at the price offered to them.*

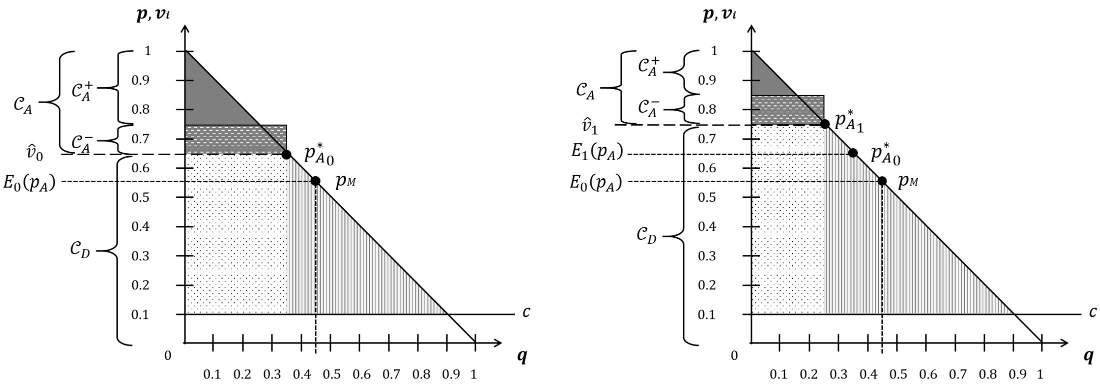
2. *If consumers have a level of strategic sophistication of $k > \bar{k} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil$, there are multiple sophistication-k equilibria with the following characteristics:*

   - *Consumers form the k-beliefs $\mathbb{E}_{\bar{k}+}(p_D) = p_i^*$ and $\mathbb{E}_{\bar{k}+}(p_A) \in [1 - s, 1]$.*
   - *No consumer anonymizes as $\hat{v}_{\bar{k}+} \in [1, 1 + s]$ and hence $v_i \leq \hat{v}_{\bar{k}+}$ for all v_i, leading to the sets $\mathcal{C}_D = [0, 1]$ and $\mathcal{C}_A = \emptyset$.*
   - *The seller forms the k+1-belief that $\mathcal{C}_D = [0, 1]$ and $\mathcal{C}_A = \emptyset$.*
   - *If $k < \infty$, the seller sets $p_i^* = \max\{v_i, c\}$ and $p_{A_k}^* \in [0, 1]$.*
   - *If $k = \infty$, the seller sets $p_i^* = \max\{v_i, c\}$ and $p_{A_\infty}^* = \mathbb{E}_\infty(p_A) \in [1 - s, 1]$.*
   - *All consumers in $\mathcal{C}_D$ with $v_i \geq c$ buy the product at the price offered to them.*
   - *No consumer buys the product via channel A.*

In the sophistication-k equilibrium captured by the first case consumers with a relatively high valuation ($v_i > \hat{v}_k$) choose the anonymous channel A, consumers with relatively low valuation ($v_i \leq \hat{v}_k$) choose the direct channel D and are perfectly price discriminated against. It is worth noting that consumers with very low valuation ($v_i < p_M$) choose the direct channel D irrespectively of k and s as they cannot possibly hope to get a uniform price that is affordable to them via channel A. Those consumers are the ones that are not served in monopolistic markets without possibility for price discrimination. Note further that the multiplicity of equilibria in the second case is only due to the multiplicity of possible beliefs about the off-equilibrium path, but all equilibria lead to the same equilibrium behavior, where no consumer anonymizes and the seller charges individualized prices p_i^* to everyone.

3 Welfare

As we have shown, different levels of consumer sophistication k lead to different anonymization behavior, which has consequences for consumer surplus (CS), profits (π), and total welfare (W). We will first take a look at consumer surplus and profits for both channels separately. Total welfare, for which we employ the customary definition, $W = CS + \pi$, and hence abstract from preferences by a social planner (or policy-maker) for either side of the market, will only be included in our final aggregate analysis. Throughout the entire section, though, Figure 2 might serve as a visualization of the different sets and quantities and illustrates the effects of an increase in k when comparing Figure 2a and Figure 2b. In the comparative statics analysis of changes in consumer sophistication the discreteness of k is taken into account by calculating changes as differences rather than differentials. Additionally, due to the potential non-linearity when increasing k from $\bar{k}-1$ to $\bar{k}$, these differences only hold for $k+1<\bar{k}$.[15]



(a) $k=0$ (b) $k=1$

Figure 2: Welfare analysis with parameters $v_i \sim \mathcal{U}[0,1]$, $c=0.1$, $s=0.1$

3.1 Channel D

Consumer Surplus and Profits in Channel D

As the seller engages in perfect price discrimination for consumers in $\mathcal{C}_D$, it is clear that

$$CS_{D_k}=0, \tag{18}$$

whereas the seller appropriates the entire surplus in channel D as profits, given by

$$\pi_{D_k}=\frac{(\hat{v}_k-c)^2}{2}=\frac{1}{8}(1-c)^2+\frac{1-c}{2}(k+1)s+\frac{(k+1)^2}{2}s^2, \tag{19}$$

corresponding to the vertically striped (lower right) triangle in Figure 2.

[15] Recall that $\bar{k}$ is usually the result of rounding (unless $\frac{1-c}{2}-1\in\mathbb{Z}_0^+$) and hence the last change in the composition of $\mathcal{C}_A$ and $\mathcal{C}_D$ is usually of different size than s, leading to similar but not identical changes. When increasing consumer sophistication from $\bar{k}-1$ to $\bar{k}$, the increase of $\mathcal{C}_D$ is bounded from above by s as *all remaining* consumers switch to channel D. Introducing separate cases in all difference equations is avoided for legibility, but addressed in the text where necessary.

Comparative Statics for k in Channel D
Recalling that $\mathcal{C}_D = [0, \hat{v}_k]$ and $\hat{v} = p_M + (k+1)s$, we note first that increasing k to $k+1$ raises $\hat{v}$ and hence increases the size of $\mathcal{C}_D = [0, \hat{v}_k]$. Letting $\Delta CS_{D_k} \equiv CS_{D_{k+1}} - CS_{D_k}$ and $\Delta\pi_{D_k} \equiv \pi_{D_{k+1}} - \pi_{D_k}$ denote the effects of increasing consumer sophistication on consumer surplus and profits in channel D, it can be shown that:

$$\text{For } k < \bar{k} - 1:$$

$$\Delta CS_{D_k} = 0, \tag{20}$$

$$\Delta\pi_{D_k} = (\hat{v}_{k+1} - c)s - \frac{s^2}{2} = \frac{1-c}{2}s + \frac{2k+3}{2}s^2. \tag{21}$$

Due to perfect price discrimination, consumer surplus in channel D, unsurprisingly, does not change when consumers become more strategically sophisticated. Profits in channel D, on the contrary, increase because the group of consumers which the seller can perfectly discriminate, $\mathcal{C}_D$, grows. This can also be seen by comparing Figure 2a and Figure 2b where the larger bracket along the vertical axis shows the increasing size of channel D and the larger striped triangle depicts the increase in profits that comes along with it. Growth of π_D continues when increasing consumer sophistication from $\bar{k} - 1$ to $\bar{k}$ (and is bounded from above by the expression in Equation (19)) and comes to a halt from there onwards as $\mathcal{C}_D$ already encompasses all consumers in the market.

Lemma 5 (Effects of Changing Consumer Sophistication (Channel D)) *Raising the level of strategic sophistication of consumers from k to $k+1$ increases the usage of channel D for all $k < \bar{k}$ (and is maximal for $k \geq \bar{k}$). Consumer surplus in channel D is nil (CS_{D_k}=0) and independent of k (ΔCS_{D_k}=0). The seller's profits from channel D are positive ($\pi_D > 0$) and increasing in k for all $k < \bar{k}$ (and maximal for $k \geq \bar{k}$).*

3.2 Channel A

Consumer Surplus and Profits in Channel A
In channel A, consumer surplus consists of two parts: the benefit from consumption of the good after the transaction at Stage 3 (denoted by $CS^+_{A_k}$) and the cost of anonymization incurred at Stage 1 (denoted by $CS^-_{A_k}$):

$$CS^+_{A_k} = \frac{(1-\hat{v}_k)^2}{2} = \frac{(1-c)^2}{8} - \frac{(1-c)}{2}(k+1)s + (k+1)^2 s^2, \tag{22}$$

$$CS^-_{A_k} = (1-\hat{v}_k)s = \frac{1-c}{2}s - (k+1)s^2. \tag{23}$$

In Figure 2, $CS^+_{A_k}$ corresponds to the solid grey (upper) triangle, whereas the dashed rectangle that partially overlaps this triangle represents the term $CS^-_{A_k}$. Net consumer surplus ($CS_{A_k} \equiv CS^+_{A_k} - CS^-_{A_k}$) in channel A then amounts to:

$$CS_{A_k} = \frac{(1-\hat{v}_k)^2}{2} - (1-\hat{v}_k)s = \frac{1}{8}(1-c)^2 - \frac{1-c}{2}(k+2)s + \frac{(k+1)(k+3)}{2}s^2. \quad (24)$$

Additionally, note that only some consumers in channel A end up with positive net surplus (denoted by $\mathcal{C}^+_A = [\hat{v}_k + s, 1]$)), whereas others end up with negative net surplus (denoted by $\mathcal{C}^-_A = (\hat{v}_k, \hat{v}_k + s)$).[16] Both sets are indicated along the vertical axis of Figure 2. The seller's profits in channel A correspond to the dotted white rectangle in Figure 2 and are given by

$$\pi_{A_k} = (1-\hat{v}_k)(\hat{v}_k - c) = \frac{1}{4}(1-c)^2 - (k+1)^2 s^2. \quad (25)$$

Comparative Statics for k in Channel A

Recalling that $\mathcal{C}_A = (\hat{v}_k, 1]$ and $\hat{v}_k = p_M + (k+1)s$, we note first that increasing k to $k+1$ raises $\hat{v}_k$ and hence decreases the size of $\mathcal{C}_A = (\hat{v}_k, 1]$. Letting $\Delta CS_{A_k} \equiv CS_{A_{k+1}} - CS_{A_k}$ and $\Delta\pi_{A_k} \equiv \pi_{AD_{k+1}} - \pi_{A_k}$ denote the effects of increasing consumer sophistication on consumer surplus and profits in channel A, it can be shown that:

For $k < \bar{k} - 1$:

$$\Delta CS_{A_k} = -\left((1-\hat{v}_{k+1})\,s + \frac{s^2}{2}\right) + s^2 = -\frac{1-c}{2}s + \frac{2k+5}{2}s^2, \quad (26)$$

$$\Delta\pi_{A_k} = (1-\hat{v}_{k+1})s - (\hat{v}_k - c)s = -(2k+3)s^2. \quad (27)$$

While the first term in Equation (26) stems from the reduction of consumer surplus from the transaction of the good at Stage 3, the second term stems from the gain from fewer consumers incurring the up-front anonymization cost. In Figure 2, the first effect is represented by the shrinking area of the dark grey triangle, and the second effect by the shrinking dashed rectangle.[17] Which of these effects dominates, determines whether consumer surplus in channel A increases or decreases. Denoting the threshold level of

[16] We include the consumer with a 0 net surplus in the set $\mathcal{C}^+_A$.

[17] The dark grey triangle shrinks by a trapezoid composed of the rectangle of area $(1-\hat{v}_{k+1})s$ and the triangle of area $\frac{s^2}{2}$, whereas the dashed rectangle has height s and shrinks in width by s, making for a decrease in area of s^2.

consumer sophistication where consumer surplus stops decreasing by $\bar{k}_{\Delta CS}$, we define:

$$\bar{k}_{\Delta CS} \equiv min\{k \in \mathbb{Z}_0^+ | \Delta CS_{A_k} \geq 0\}. \tag{28}$$

Using Equation (26) in (28), solving for $\bar{k}_{\Delta CS}$, and following the same line of reasoning to deal with the discreteness of k as before yields:

$$\bar{k}_{\Delta CS} \geq \frac{1-c}{2s} - \frac{5}{2} \Rightarrow \bar{k}_{\Delta CS} = \left\lceil \frac{1-c}{2s} - \frac{5}{2} \right\rceil. \tag{29}$$

To get a better impression of the location of this threshold, recall that $\bar{k} = \lceil \frac{1-c}{2s} - 1 \rceil$ and therefore

$$\bar{k} - \bar{k}_{\Delta CS} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil - \left\lceil \frac{1-c}{2s} - \frac{5}{2} \right\rceil = \left\lceil \frac{1-c}{2s} \right\rceil - \left\lceil \frac{1-c}{2s} - \frac{1}{2} \right\rceil + 1 \in \{1, 2\}, \tag{30}$$

which reveals that consumer surplus stops decreasing already one or two levels of sophistication before channel A breaks down. While this seems counterintuitive at first, it is helpful to recall that $\mathcal{C}_A = \mathcal{C}_A^- \cup \mathcal{C}_A^+$ and that $\mathcal{C}_A^-$ is situated below $\mathcal{C}_A^+$. Hence, as k increases, $\mathcal{C}_A^+$ seizes to contain consumers before $\mathcal{C}_A^-$ does, which in turn means that consumer surplus eventually turns negative. Denoting the additional thresholds $\bar{k}_{CS}$, where consumer surplus turns negative, and $\bar{k}_{\mathcal{C}_A^+}$, where no consumer in channel A makes a net surplus from the transaction anymore, we define:

$$\bar{k}_{CS} \equiv min\{k \in \mathbb{Z}_0^+ | CS_{A_k} = 0\}, \tag{31}$$

$$\bar{k}_{\mathcal{C}_A^+} \equiv min\{k \in \mathbb{Z}_0^+ | \mathcal{C}_A^+ = \emptyset\}. \tag{32}$$

Using Equation (24) in (31) and the definition of $\mathcal{C}_A^- = (\hat{v}_k + s, 1]$ in Equation (32), solving for the respective thresholds and following the same line of reasoning to deal with the discreteness of k as before yields:

$$\bar{k}_{CS} \geq \frac{1-c}{2s} - 3 \Rightarrow \bar{k}_{CS} = \left\lceil \frac{1-c}{2s} - 3 \right\rceil, \tag{33}$$

$$\bar{k}_{\mathcal{C}_A^+} \geq \frac{1-c}{2s} - 2 \Rightarrow \bar{k}_{\mathcal{C}_A^+} = \left\lceil \frac{1-c}{2s} - 2 \right\rceil. \tag{34}$$

Similarly, these thresholds can be put in relation to the level of sophistication at which the market for anonymization breaks down:

$$\bar{k} - \bar{k}_{CS} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil - \left\lceil \frac{1-c}{2s} - 3 \right\rceil = \left\lceil \frac{1-c}{2s} \right\rceil - \left\lceil \frac{1-c}{2s} \right\rceil + 2 = 2, \tag{35}$$

$$\bar{k} - \bar{k}_{\mathcal{C}_A^+} = \left\lceil \frac{1-c}{2s} - 1 \right\rceil - \left\lceil \frac{1-c}{2s} - 2 \right\rceil = \left\lceil \frac{1-c}{2s} \right\rceil - \left\lceil \frac{1-c}{2s} \right\rceil + 1 = 1. \tag{36}$$

Equation (35) shows that the combined cost of anonymization incurred by all consumers in $\mathcal{C}_A$ outweighs the combined surplus from the transaction of the good at the penultimate level before the breakdown of channel A, while at the last level before the breakdown of channel A there are no consumers in channel A anymore that make a net surplus, as Equation (36) shows. Taken together, they provide the two options derived in Equation (30) for the level of sophistication at which consumer surplus stops decreasing. Hence, we can resolve the counterintuitive result that consumer surplus can stop decreasing already at $\bar{k} - 2$ by having shown that this is only possible because consumer surplus is 0, at best, at this point and will be negative at $\bar{k} - 1$ the latest. Due to the discreteness of k, the minimum might be attained at either level (indicated by the result of Equation (30)). In any case, raising the level of strategic sophistication from $\bar{k} - 1$ to $\bar{k}$ leads to an increase in consumer surplus as channel A remains unused and consumer surplus jumps to 0 as all consumers are being perfectly price discriminated in channel D.

To summarize our discussion of consumer surplus in somewhat more plain terms: we have shown that consumers lose surplus the more strategically sophisticated they become until everyone "gives in" to the seller's price discrimination practices in the direct channel D.

Profits in channel A, however, are generally decreasing in consumer sophistication, as Equation (27) shows. Equation (27) further highlights that the gains at the intensive margin (charging a higher price to all remaining consumers in channel A), given by $(1 - \hat{v}_{k+1})s$, are outweighed by the loss at the extensive margin (the loss of revenue from consumers that switched to channel D), given by $(\hat{v}_k - c)s$. This result, however, should come as no surprise given the general discussion about the seller's profits from channel A (cf. Figure 1 in Section 1). Contrary to consumer surplus, there are no thresholds determining a change in this process for profits in channel A as they continue decreasing until channel A is not used by any consumer.

Lemma 6 (Effects of Changing Consumer Sophistication (Channel A)) *Raising the level of strategic sophistication of consumers from k to $k+1$ decreases the usage of channel A for all $k < \bar{k}$ (and is nil for $k \geq \bar{k}$). Consumer surplus (CS_A) de-*

creases for all $k < \bar{k}_{\Delta CS} = \left\lceil \frac{1-c}{2s} - \frac{5}{2} \right\rceil \in \{\bar{k} - 2; \bar{k} - 1\}$ and becomes non-positive at $\bar{k}_{CS} = \left\lceil \frac{1-c}{2s} - 3 \right\rceil = \bar{k} - 2$. Additionally, at $\bar{k}_{C_A^+} = \left\lceil \frac{1-c}{2s} - 2 \right\rceil = \bar{k} - 1$ all consumers in channel A incur a net loss. The seller's profits from channel A (π_A) are positive but decreasing in k for all $k < \bar{k}$ (and nil for all $k \geq \bar{k}$).

3.3 Aggregate Market (Channel A & Channel D)

Consumer Surplus, Profits, and Welfare

After the separate analysis of both channels we now return to the bigger picture that consolidates the different effects and allows for an overall welfare analysis. Defining $CS_k \equiv CS_{D_k} + CS_{A_k}$, $\pi_k \equiv \pi_{D_k} + \pi_{A_k}$, and $W_k \equiv CS_k + \pi_k$ leads to the following results (combining Equations (18) and (24) to (37), Equations (19) and (25) to (38), and, ultimately, Equations (37) and (38) to (39):

$$CS_k = \frac{(1-\hat{v}_k)^2}{2} - (1-\hat{v}_k)s = \frac{1}{8}(1-c)^2 - \frac{1-c}{2}(k+2)s + \frac{k^2+4k+3}{2}s^2, \quad (37)$$

$$\pi_k = \frac{(\hat{v}_k - c)^2}{2} + (1-\hat{v}_k)(\hat{v}_k - c) = \frac{3}{8}(1-c)^2 + \frac{1-c}{2}(k+1)s - \frac{k^2+2k+1}{2}s^2, \quad (38)$$

$$W_k = \frac{(1-c)^2}{2} - (1-\hat{v}_k)s = \frac{1}{2}(1-c)^2 - \frac{1-c}{2}s + (k+1)s^2. \quad (39)$$

Like total consumer surplus and total profits, total welfare depends on the strategic level of sophistication of consumers and can be identified graphically in Figure 2, too.[18] The first term, $\frac{1}{2}(1-c)^2$, corresponds to the whole area between the demand curve and the marginal cost curve in Figure 1, while the second term, $(1-\hat{v}_k)s$, corresponds to the dashed rectangle. Although the market outcome of Stage 3 is efficient, because every consumer with a valuation $v_i \geq c$ buys the product, this shows that total welfare is reduced by the losses stemming from consumers' anonymization behavior as long as $\hat{v}_k < 1$ or, equivalently, $k < \bar{k}$. For any $k \geq \bar{k}$, a fully efficient outcome ensues, showing that the nested case of assuming unlimited strategic sophistication of consumers ($k = \infty$) is *sufficient* to ensure efficiency but *not necessary*.

Comparative Statics for k for the Aggregate Market

Similarly as for the two channels before, we derive the effects on the aggregated quantities

[18] An in-depth discussion of the terms of Equations (37) and (38) can be found in the respective channel's discussions.

as differences due to the discrete nature of changes in consumer sophistication:

For $k < \bar{k} - 1$:

$$
\begin{aligned}
\Delta CS_k \equiv \quad CS_{k+1} - CS_k &= -(1-\hat{v}_{k+1})\,s + \frac{s^2}{2} = -\frac{1-c}{2}s + \frac{2k+5}{2}s^2, && (40)\\
\Delta \pi_k \equiv \quad \pi_{k+1} - \pi_k &= (1-\hat{v}_k)\,s + \frac{s^2}{2} = \frac{1-c}{2}s + \frac{2k+3}{2}s^2, && (41)\\
\Delta W_k \equiv \quad W_{k+1} - W_k &= s^2 = s^2. && (42)
\end{aligned}
$$

Since consumer surplus from channel D was equal to zero independent of k, the effect of changing k on aggregate consumer welfare is identical to the already identified effect in channel A, i.e. decreasing as k increases until a certain threshold, $\bar{k}_{\Delta CS}$ is reached. Recognizing the similarity in Equation (41), we define an additional threshold level of consumer sophistication where profits stop increasing $\bar{k}_\pi$:

$$\bar{k}_\pi \equiv min\{k \in \mathbb{Z}_0^+ | \Delta\pi_k \leq 0\}. \tag{43}$$

Substituting Equation (41) in (43), solving for the threshold level, and again following the same line of reasoning to deal with the discreteness of k as before yields:

$$\bar{k}_\pi \leq \frac{1-c}{2s} - \frac{3}{2} \Rightarrow \bar{k}_\pi = \left\lceil \frac{1-c}{2s} - \frac{3}{2} \right\rceil. \tag{44}$$

As before, this threshold is compared to the level of sophistication at which the market for anonymization breaks down:

$$\bar{k} - \bar{k}_\pi = \left\lceil \frac{1-c}{2s} - 1 \right\rceil - \left\lceil \frac{1-c}{2s} - \frac{3}{2} \right\rceil = \left\lceil \frac{1-c}{2s} \right\rceil - \left\lceil \frac{1-c}{2s} - \frac{1}{2} \right\rceil \in \{0, 1\}. \tag{45}$$

Equation (45) indicates that profits stop increasing either the last level before the breakdown of channel A or when this happens. Recalling, however, that all comparative statics difference equations (and hence also Equation (41) which we used in deriving $\bar{k}_\pi$) are only applicable to $k \leq \bar{k} - 2$, we have to examine this case closer since $\bar{k}_\pi \in \{\bar{k}-1, \bar{k}\}$. Further recalling that $\mathcal{C}_D$ increases until $k = \bar{k}$ and that the seller appropriates all surplus from any consumer in channel D while he only receives a share of the surplus generated from the transaction when selling to consumers in channel A, it is straightforward to conclude that profits are still increasing when consumers' sophistication changes from $\bar{k} - 1$ to $\bar{k}$.

Hence, we have to adjust Equations (44) and (45) to (46) and (47), respectively:

$$\bar{k}_\pi = \left\lceil \frac{1-c}{2s} - 1 \right\rceil, \tag{46}$$

$$\bar{k} - \bar{k}_\pi = 0. \tag{47}$$

Although increasing k has negative effects on consumer surplus and positive effects on profits, welfare is generally increasing in k as Equation (42) shows (and it, too, does so including the last change from $\bar{k}-1$ to $\bar{k}$). A threshold cannot even be determined as the change is independent of k). This result is, of course, driven by the fact that increasing the level of sophistication leads to fewer anonymized consumers, corresponding to smaller cost of anonymization, all the while the surplus from the transaction of the good stays constant at the maximum due to perfect price discrimination in channel D (raising k simply shifts the surplus from consumers to the seller).

Combining the insights of the previous Lemmas, we summarize the above analysis in the following propositions.

Proposition 2 (Consumer Sophistication and Welfare) *For any non-prohibitively high cost of anonymization $s > 0$ and cost of production $c > 0$ and any level of consumer sophistication k, aggregated consumer surplus (CS_k), profits (π_k), and welfare (W_k) exhibit the following characteristics:*

- *$CS_k > 0$ for $k < \bar{k}_{CS}$, $CS_k \leq 0$ for $\bar{k}_{CS} \leq k < \bar{k}$, and $CS_k = 0$ for $k \geq \bar{k}$, where $\bar{k}_{CS} = \left\lceil \frac{1-c}{2s} - 3 \right\rceil$ and $\bar{k} - \bar{k}_{CS} = 2$.*
- *$\pi_k > 0$ for $k < \bar{k}$, and $\pi_k = W_k$ for $k \geq \bar{k}$.*
- *$W_k > 0$ for $k < \bar{k}$, and $W_k = W^*$ for $k \geq \bar{k}$, where W^* is the first-best outcome.*

Proposition 3 (Effects of Changing Consumer Sophistication) *Raising the level of strategic sophistication of consumers from k to $k+1$ has the following effects on consumer surplus, profits, and welfare (ceteris paribus):*

- *$\Delta CS_k < 0$ for $k < \bar{k}_{\Delta CS}$, $\Delta CS_k \geq 0$ for $\bar{k}_{\Delta CS} \leq k < \bar{k}$, and $\Delta CS_k = 0$ for $k \geq \bar{k}$, where $\bar{k}_{\Delta CS} = \left\lceil \frac{1-c}{2s} - \frac{5}{2} \right\rceil$ and $\bar{k} - \bar{k}_{\Delta CS} \in \{1, 2\}$.*
- *$\Delta\pi_k > 0$ for $k < \bar{k}$, and $\Delta\pi_k = 0$ for $k \geq \bar{k}$.*
- *$\Delta W_k > 0$ for $k < \bar{k}$, and $\Delta W_k = 0$ for $k \geq \bar{k}$.*

Corollary 1 (Positive and Negative Individual Surplus) *As long as consumers are not* too *sophisticated* ($k < \bar{k}_{\mathcal{C}_A^+} = \left\lceil \frac{1-c}{2s} - 2 \right\rceil = \bar{k} - 1$)*, some consumers who anonymize themselves (those in* $\mathcal{C}_A^+$*) end up with positive net surplus, whereas others (those in* $\mathcal{C}_A^-$*) end up with negative net surplus.*

Proposition 3 predicts that (except for boundary cases) the strategic sophistication of consumers will work to their disadvantage at an aggregated level and, if too sophisticated, can break down the market for anonymous shopping. By contrast, the seller benefits from this stepwise breakdown, a development that would also be appreciated by a total welfare maximizer. The reason for this preference is, interestingly, not based on allocation: Due to perfect price discrimination in the direct channel, all consumers with a valuation for the product above its marginal cost of production get the product independent of the existence of the anonymous channel. Instead, given that big data technologies driving channel D are already in place, it is inefficient to incur the additional cost of anonymization. The smaller channel A, the smaller this inefficiency.

Corollary 1 zooms into the set of consumers who choose to invest in their anonymization. See Figure 2, where three groups of consumers have been distinguished: $\mathcal{C}_D$, $\mathcal{C}_A^-$, and $\mathcal{C}_A^+$. While the first denotes those consumers who chose channel D, the superscript at the two remaining groups distinguishes those consumers in channel A who make a net loss (because they do not predict the seller's incentive to increase the price by s fully) from those ending up with a net benefit of the whole transaction.

Comparative Statics for s

Apart from the effects attributable to changes in strategic sophistication of consumers, we also analyze the effects of changes in the cost of anonymization. This analysis may inform whether policy efforts to make anonymizing techniques available at lower cost are desirable. Before delving into the effects on consumer surplus, profits and welfare, it is useful to first identify the threshold when anonymization cost becomes prohibitively high for channel A to be used by any consumer (the equivalent of $\bar{k}$) as our analysis will be limited by this upper bound. Denoting this threshold by $\bar{s}$, we define:

$$\bar{s} \equiv min\{s \in \mathbb{R}_0^+ | \mathcal{C}_A = \emptyset\}. \tag{48}$$

Recalling that $\mathcal{C}_A = (\hat{v}_k; 1]$ and $\hat{v}_k = \frac{1+c}{2} + (k+1)s$, yields:

$$\mathcal{C}_A = \emptyset \Leftrightarrow \hat{v}_k \geq 1 \Leftrightarrow s \geq \frac{1-c}{2}\frac{1}{(k+1)}, \tag{49}$$

$$\Rightarrow \bar{s} = \frac{1-c}{2}\frac{1}{(k+1)}. \tag{50}$$

Since s, contrary to k, is a continuous variable, we do not need to take further special cases into account. The continuous nature of s further implies that the effects on consumer surplus, profits, and welfare are therefore found by differentials (instead of differences) using Equations (37), (38), and (39):

$$\frac{\partial CS_k}{\partial s} = -\frac{1-c}{2}(k+2) + (k+1)(k+3)s = (k+1)s - (k+2)(1-\hat{v}_k), \tag{51}$$

$$\frac{\partial \pi_k}{\partial s} = \frac{1-c}{2}(k+1) - (k+1)^2 s = (k+1)(1-\hat{v}_k), \tag{52}$$

$$\frac{\partial W}{\partial s} = -\frac{1-c}{2} + 2(k+1)s = (k+1)s - (1-\hat{v}_k). \tag{53}$$

Defining thresholds in a similar way as in our analysis of the effects of raising consumer sophistication and limiting the analysis to non-prohibitive cost of anonymization, yields:

For $s \leq \bar{s}$:

$$\frac{\partial CS_k}{\partial s} \begin{cases} < 0 & \text{if } s < \frac{1-c}{2}\frac{k+2}{(k+1)(k+3)} \equiv \bar{s}_{CS}, \\ \geq 0 & \text{if } s \geq \frac{1-c}{2}\frac{k+2}{(k+1)(k+3)} \equiv \bar{s}_{CS}, \end{cases} \tag{54}$$

$$\frac{\partial \pi_k}{\partial s} \begin{cases} > 0 & \text{if } s < \frac{1-c}{2}\frac{1}{k+1} \equiv \bar{s}_{\pi}, \\ \leq 0 & \text{if } s \geq \frac{1-c}{2}\frac{1}{k+1} \equiv \bar{s}_{\pi}, \end{cases} \tag{55}$$

$$\frac{\partial W_k}{\partial s} \begin{cases} < 0 & \text{if } s < \frac{1-c}{2}\frac{1}{2k+2} \equiv \bar{s}_{W}, \\ \geq 0 & \text{if } s \geq \frac{1-c}{2}\frac{1}{2k+2} \equiv \bar{s}_{W}. \end{cases} \tag{56}$$

It can further be shown that

$$\bar{s} = \bar{s}_{\pi} > \bar{s}_{CS} > \bar{s}_{W}, \tag{57}$$

which reveals that the seller's profits increase in s until the prohibitive level $\bar{s}$ is reached. As it becomes more costly to anonymize, more consumers will choose channel D instead of channel A, where the seller appropriates their entire valuation as profit. The effects on consumer surplus and total welfare on the other hand, are ambiguous depending on the initial level of s. This is due to the changing effects of raising s on the composition of $\mathcal{C}_A$: At first, $\mathcal{C}_A^-$ increases in size as s increases, but when there are no consumers $\mathcal{C}_A^+$ anymore, a further increase will reduce the size of $\mathcal{C}_A^-$ again. Taking the respective second derivatives provides further insights and yields:

$$\frac{\partial^2 CS_k}{\partial s^2} = (k+1)(k+3) > 0, \tag{58}$$

$$\frac{\partial^2 W_k}{\partial s^2} = 2(k+1) \qquad > 0. \tag{59}$$

Thus, consumer surplus as well as total welfare are convex in s, implying that both reach their respective maximum at either extreme value of $s = 0$ or $s = \bar{s}$. Including profits for completeness and substituting $s = 0$ and $s = \bar{s}$ in Equations (37), (38), and (39) yields:

$$CS_k(s=0) = \frac{1}{8}(1-c)^2, \qquad CS_k(s=\bar{s}) = 0, \tag{60}$$

$$\pi_k(s=0) = \frac{3}{8}(1-c)^2, \qquad \pi_k(s=\bar{s}) = \frac{1}{2}(1-c)^2, \tag{61}$$

$$W_k(s=0) = \frac{1}{2}(1-c)^2, \qquad W_k(s=\bar{s}) = \frac{1}{2}(1-c)^2. \tag{62}$$

Within the equilibrium framework of the model so far, we did not consider $s = 0$. In this special case, the difference between consumers' expectations and seller's optimal price disappears. Hence, failing to anticipate the sunk cost nature of s does not matter anymore and no consumer in channel A incurs a net loss. As the seller optimally sets the price that consumers expect, there is no change in expectations with increasing consumer sophistication. The resulting equilbrium behavior is summarized in Lemma 7.

Lemma 7 (Sophistication-k Equilibrium with Costless Anonymization) *With costless anonymization, $s = 0$, and for any non-prohibitive cost of production $c > 0$, there is a unique sophistication-k equilibrium with the following characteristics:*

- *Consumers form the k-beliefs $\mathbb{E}_k(p_D) = p_i^*$ and $\mathbb{E}_k(p_A) = p_M = \frac{1+c}{2}$.*
- *Consumers anonymize if, and only if, $v_i > \hat{v}_k = p_M$, separating into the sets $\mathcal{C}_D = [0, p_M]$ and $\mathcal{C}_A = (p_M, 1]$.*
- *The seller forms the k+1-belief that $\mathcal{C}_D = [0, p_M]$ and $\mathcal{C}_A = (p_M, 1]$.*
- *The seller sets $p_i^* = \max\{v_i, c\}$ and $p_{A_0}^* = p_M$.*
- *All consumers in $\mathcal{C}_D$ with $v_i \geq c$ buy the product at the price offered to them.*

- *All consumers in $\mathcal{C}_A$ buy the product at the price offered to them.*

Additionally, channel A does not break down for any level of consumer sophistication (including $k = \infty$) and an efficient outcome ensues irrespective of k. Including this boundary case $s = 0$ in our analysis of the effects of anonymization cost, the comparative statics analysis for s is summarized in Proposition 4 and visualized in Figure 3.

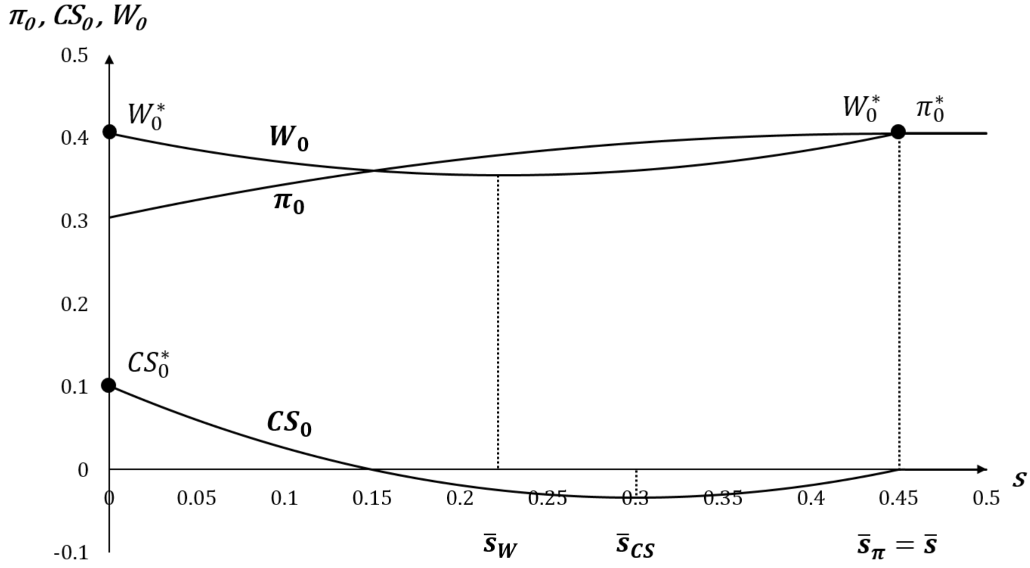


Figure 3: Consumer surplus, profits and welfare as functions of s
with parameters $v_i \sim \mathcal{U}[0, 1]$, $c = 0.1$, $k = 0$

Proposition 4 (Anonymization Cost and Welfare) *For any non-prohibitively high level of consumer sophistication $k < \bar{k}$ and cost of production $c > 0$, anonymization is prohibitively costly for $s \geq \bar{s} = \frac{1-c}{2}\frac{1}{k+1}$. Then, channel A remains unused. As long as channel A is used, aggregated consumer surplus (CS_k), profits (π_k), and welfare (W_k) exhibit the following characteristics:*

- *CS_k is maximal at $s = 0$, decreases in s to its minimum (which is negative) at $s = \bar{s}_{CS}$, then increases in s back to 0 at $s = \bar{s}$.*

- *π_k is minimal at $s = 0$ and increases in s to its maximum at $s = \bar{s}_{CS} = \bar{s}$.*

- *W_k is maximal at $s = 0$, decreases in s to its minimum at $s = \bar{s}_W$, then increases in s to another maximum at $s = \bar{s}$. Both maxima lead to the first-best outcome.*

Proposition 4 shows that higher cost of anonymization are negative for consumers despite the fact that consumer surplus increases in s for relatively high values, which becomes apparent from the fact that consumer surplus is maximal when anonymization is costless. The seller, on the other hand, unambiguously benefits from higher cost of anonymization and would prefer a prohibitively high cost of $s = \bar{s}$ as he maximizes his profits when he can perfectly discriminate and extract the entire surplus from all consumers. A total welfare maximizer, focusing on the welfare-deteriorating role of s, can choose either extreme to prevent consumers from incurring the cost: To achieve an efficient outcome, anonymization should be costless ($s = 0$) or prohibitively costly ($s = \bar{s}$). Both options are welfare-maximizing, but lead to different allocations of the surplus generated by the market. Note that, while the seller makes positive profits in either welfare-maximizing scenario, consumers only receive positive surplus when $s = 0$.

4 Alternative Model Specifications

The model we have presented so far builds on several assumptions, especially concerning the belief formation of consumers, that warrant further inquiry as to how robust the model is to changes in these assumptions.

4.1 Beliefs of "naïve" Consumers

In our model we have assumed that "naïve" consumers will expect the price in channel A to be equal to the unconditional monopoly price p_M. Many other applications of level-k thinking employ a random distribution as a starting point for players with $k = 0$. If the "naïve" consumers in our model were to make their anonymization decision randomly rather than to react to a belief of facing the unconditional monopoly price, the seller would correctly infer this and set the price accordingly. Depending on the location of the expected valuation of anonymized consumers, three cases can be distinguished:

(a) The expected valuation of anonymized consumers is *below* the unconditional monopoly price, i.e. $E(v_i|i \in \mathcal{C}_A) < p_M$.

(b) The expected valuation of anonymized consumers is *equal to* the unconditional monopoly price, i.e. $E(v_i|i \in \mathcal{C}_A) = p_M$.

(c) The expected valuation of anonymized consumers is *above* the unconditional monopoly price, i.e. $E(v_i|i \in \mathcal{C}_A) > p_M$.

These cases are equivalent to the cases for the anonymization threshold $\hat{v}$ in section 2. As discussed already there, the seller's best response in cases (a) and (b) is to charge the unconditional monopoly price p_M. This would require our analysis to include one additional first step of strategic iteration, such that consumers would expect the unconditional monopoly price for channel A if they had a level of strategic sophistication of $k = 1$. The size of this step, however, depends on the difference between the expected valuation and the unconditional monopoly price interrupting the uniformity of adjustments. In case (c), however, the seller's best response is to charge a price equal to $E(v_i|i \in \mathcal{C}_A) + s$, essentially responding in the same way as before by increasing the price by s above the cutoff valuation. Depending on the exact distance from p_M, this would reduce the number of steps until the complete breakdown of channel A, but not change the underlying mechanism of iterations from there onwards. Hence, while the choice of p_M as a starting point for our analysis pins the analysis to a particular point, it does not crucially affect the model. In the special case of costless anonymization, though, changing the belief of naïve consumers has a larger influence. As the iterative process is suspended, expectations do not change after the initial change from $k = 0$ to $k = 1$, which only changes the seller's best response in case (a), but does so in the same fashion as discussed above for $s > 0$. This allows for any price $p_A \in [p_M, 1]$ to become the constant threshold valuation $\hat{v}$, which in turn shifts the surplus allocation. Thus, while the resulting equilibrium is not necessarily efficient for $k = 0$ anymore, it is for any $k \geq 1$ and hence does not constitute a crucial departure from our model either.

4.2 Heterogeneous Cost of Anonymization

Our model further assumes that all consumers find it equally costly to use the anonymous channel A. However, it is easy to imagine that some people might find it less cumbersome to discover and make use of privacy-protecting technologies such as deleting cookies, activating "do not track" browser options or installing various plugins.[19]. Additionally, heterogeneity in s can stem from differing exogenous tastes for privacy in the consumer population which would then reduce the experienced disutility of a possibly still fixed cost of using channel A. Naturally, a heterogeneous distribution of s could be seen as the result of aggregating both effects. Allowing for heterogeneous cost of anonymization implies that demand in channel A can become a stepwise function (depending on the range

[19] For instance, TOR is "free software and an open network that helps [users] defend against traffic analysis, a form of network surveillance that threatens personal freedom and privacy, confidential business activities and relationships, and state security" (`https://www.torproject.org/`). A more detailed list can be found in Sellenart's "A paranoid's toolbox for browsing the web": `http://pierre.senellart.com/talks/cerre-20160915.pdf` (Sellenart 2016, 20).

of s), and possibly non-linear for some range of valuations, which makes it a complex endeavor to solve the seller's optimization problem analytically. Numerical examples, indicate that unravelling still occurs, although in a non-linear manner. A full breakdown of channel A is no longer achieved at a finite level of sophistication, but becomes the limit case for $k = \infty$, rendering it a *necessary* condition for a fully efficient outcome. Otherwise, consumers always suffer from their limited strategic sophistication, while the seller benefits from it. Thus, the general pattern of a gradual breakdown of the anonymous channel and the corresponding welfare effects would not be altered by this additional layer of complexity.

5 Discussion and Conclusion

This paper started from the empirical observation that the technological developments that are alluded to as the "rise of big data" or "datafication" have led to asymmetries on markets for consumer goods (Mayer-Schönberger and Cukier 2013). Sellers making use of huge datasets with information on choices of large masses of consumers can tailor prices to individual characteristics and thereby appropriate a greate share of the surplus created by market transactions. On top of the sheer amount of information that is available to sellers, consumers are at a second disadvantage. They face cognitive constraints regarding strategic sophistication (Acquisti and Grossklags 2007), while the seller's data processing capabilities enable him to always respond optimally.

In this paper, we have taken these developments serious and constructed a model to study their implications on prices, consumption choices, and consumers' incentives to use anonymization technologies protecting their privacy. We have shown that under certain conditions, most notably under the assumption of imperfect strategic sophistication of consumers, a costly privacy-protective sales channel is used even if consumers do not have an explicit taste for privacy. In our model, consumers want to restore their privacy (i.e. choose channel A) solely based on their valuation of the good and their price expectation. We thereby provide a micro-foundation as to why consumers might have such a taste for privacy, to which the existence of a privacy-protective sales channel can cater.

In particular, we have studied the role of consumers' strategic sophistication and the role of the size of the anonymization cost on equilibrium behavior. In contrast to earlier work, applying a model of level-k thinking to this industrial organization context has allowed us to compare not only a population of completely naïve consumers with an unlimitedly sophisticated one but to also study the effects of gradually increasing consumer sophistication on market outcomes. Our model revealed that unlimited strategic sophistication is a sufficient but not a necessary condition for the breakdown of the anonymous sales channel if anonymization is equally costly to all consumers. Allowing for heterogeneity in anonymization cost, sources of which can be different technological savviness, but also differing preferences for privacy, would reinstate the necessity of unlimited strategic sophistication for a complete breakdown of the anonymous channel, though.

In general, the use of big data technologies by sellers improves total welfare by avoiding the dead weight loss usually associated with a monopoly: In contrast to uniform monopoly pricing, consumers with low valuations, $v_i < p_M$, can purchase the product now. This increases efficiency but not consumer surplus as the seller appropriates the entire surplus from these additional transactions. We have further shown that using the anonymous channel backfires and leads to a net loss for at least some (and under certain conditions all) anonymized consumers (forming the set $\mathcal{C}_A^-$).

Focusing on consumer sophistication, we have shown that increasing consumer sophistication leads to a reduction in consumer surplus but to an increase in profits and total welfare. Given that the level of strategic sophistication, k, may be regarded as exogenous for policy makers, our results concerning the cost of anonymization, s, might be more policy relevant.

Concerning the cost of anonymization, we have shown that consumer surplus is maximal in the extreme case of costless anonymization, $s = 0$, profits are maximal in the extreme case of anonymization being prohibitively costly, $s = \bar{s}$, whereas total welfare is maximal at either extreme $s = 0$ or $s = \bar{s}$. The two fully efficient cases differ, though, in the way in which they ensure a first-best result. In the case $s = 0$, on the one hand, consumers separate themselves at the monopoly price p_M and those with high valuation, $v_i > p_M$, anonymize for free and receive positive surplus, whereas all others choose the direct channel where they get perfectly discriminated against and are left with zero surplus. As consumers make use of the anonymous channel, efficiency is driven by this choice not imposing any cost to consumers. On the other hand, if $s = \bar{s}$ consumers do not separate themselves into two groups anymore and choose the direct channel irrespective of their valuation for the good. Hence in this case, efficiency is brought about by the fact that anonymization is simply too costly and consumers rather leave the entire surplus to the seller than to incur the cost of anonymizing.

Which of these two extremes should be favored therefore crucially depends on the objective function of a possibly intervening authority. A consumer-oriented welfare maximizer will try to eliminate anonymization cost, whereas a seller-oriented welfare maximizer will seek to increase the cost of anonymization to a prohibitively level.

A policy maker with a preference for consumer surplus could, for example, require marketers and online platforms serving as matchmakers between sellers and buyers of consumer goods to set anonymous shopping technologies as *default* and to only offer consumers to *opt in* to non-anonymous shopping (instead of today's standard, where full tracking of consumers' choices is the default and a few providers offer *opt out* technologies). This proposal is also discussed by Acquisti, et al. (2016). Thereby, those consumers who find it in their interest to reveal their true characteristics to sellers (those with low valuations for a given product) would log in to some service and receive the product for a low price, equalizing their willingness-to-pay. Consumers with higher valuation would stay with the (now default) anonymous channel and pay a higher price for the product, but still retain some surplus.

On the theory side, future research could shed light on the effects of heterogeneity in the level of strategic sophistication amongst consumers, relying on a more elaborate cognitive hierarchy model than this first attempt we undertook here. This is a complex undertaking, however, because it is not only necessary to specify a distribution of k-levels across the population of consumers (and how it might be related to their willingness-to-pay) but it also requires to specify every consumer's belief about other consumers' sophistication, as a function of that consumer's own sophistication. On a more general note, the concept of a sophistication-k equilibrium could also be applied to other market interactions where players are likely not on an equal footing when it comes to iterative strategic thinking.

As regards empirical testing of our theory, we consider it most promising to conduct laboratory experiments because subjects can be assigned certain valuations which are then known and a discriminatory pricing algorithm can actually be implemented. Moreover, laboratory experiments are less susceptible to noise in the elicitation of people's level of strategic sophistication than field experiments. Subjects then indicate their respective anonymization choice given their valuation and the cost of anonymization. The implied thresholds for anonymization correspond to a certain level of strategic sophistication according to our model, which in turn could be compared to other measures of strategic sophistication spawned from the level-k literature. Using measures that capture differing degrees of belief interactions of subjects could inform whether the current model that neglects more complex cognitive hierarchies is a fair representation of subjects' approach to such a market or whether efforts to generalize our theory are needed.

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